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Reduction of Complexity By Optimal Driving Forces *

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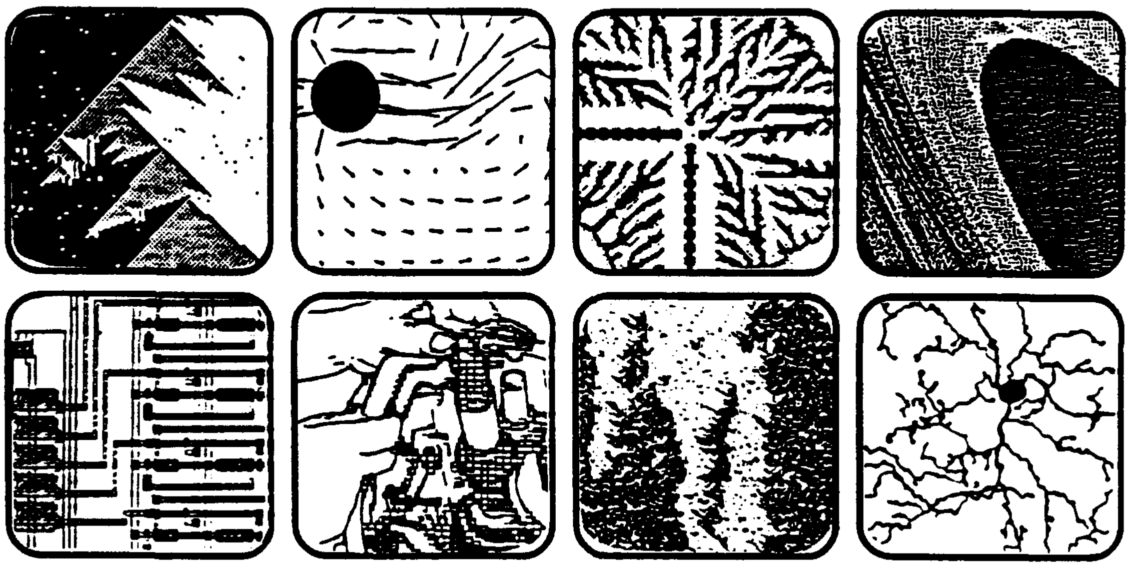

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Abstract. In general nonlinear waves are not stable in a chain of finite length. Since they have a finite lifetime, it is important to investigate the production of nonlinear waves, e.g. the production of solitons. A general feature of nonlinear waves is the amplitude frequency coupling, which causes the excitation by sinusoidal driving forces to be very inefficient. The response is usually very complex in addition. We present a method to calculate special aperiodic driving forces, which generates nonlinear waves very efficiently. The response to these driving forces is very simple.

INTRODUCTION

When nonlinear oscillator is perturbed by a sinusoidal force, the response is comparatively small in amplitude [1], and does not fulfil any well defined resonance condition [2], even when the frequency of the driving force coincides with a peak (resonance) in the power spectrum of the unperturbed system [3]. Outside the region of entrainment the response is complicated, in many cases chaotic [4]. In order to obtain large, simple and predictable response, the frequency of the driving force has to be varied in such a way, that it coincides at all amplitudes with the characteristic frequency of the oscillator [5]. Since the characteristic frequencies of nonlinear oscillators usually depends on the amplitude the optimal driving force has to be aperiodic. Recently a method to calculate those optimal driving forces has been presented [6]. We apply this method in order to calculated optimal driving forces for the creation of solitons.

CREATION OF SOLITONS BY APERIODIC DRIVING FORCES

Nonlinear waves and solitons provide good mathematical models in various fields of science [7]. In most experimental systems solitons have a long

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but finite lifetime. Therefore we investigate the creation of solitons by external perturbations. We assume that the dynamics of the experimental system can modeled by a sine Gordon equation

$$u_{xx} - u_{tt} - sin(u) = F(x,t) \tag{0.1}$$

where $u(x,t)$ is the field amplitude which depends on space x and time t and where F is an external perturbation which only depends on time and space. In order to calculate resonant driving forces we integrate according to Ref. 6 the following goal dynamics

$$w_{xx} - w_{tt} - Bsin(w) + w_t\Theta\left(|x-50|-2.5\right) = 0 \tag{0.2}$$

where B is a parameter and where Θ is Heavyside's step function. We take circular or fixed boundaries at $x = 0$ and $x = 100$. The simulation if finished at time T when $|w(x,T)| \geq \pi$. The initial conditions are $w(x,0) = .0$ and $w(50,0) = .001$. The driving force results from

$$F(x,t) = -w_t(x,t)\,\Theta\left(|x-50|-2.5\right) \tag{0.3}$$

and $F(x,t) = 0$ for $t \geq T$. The basic idea is, that if the structure of Eq. (0.1) and Eq. (0.2) are the same, i.e. $B = 1$, $u(x,t) = w(x,t)$ is a special solution of Eq. (0.1). In this case the energy transfer $P(t) = \int_0^{100} F\dot{u}dx$ is positive for all t i.e. no energy is reflected since F is proportional to w_t. Therefore the coefficient of absorption is 100%, the reaction power is zero and the perturbation is resonant. The special space dependence of F was taken in order to create solitons instead of other nonlinear waves. Fig. 1a shows the result of a numerical simulation of the response of the sin-Gordon system. For the integration we use 100 homogeneously distributed break points. The initial amplitudes of u at these break points are randomly distributed in the interval $[-10^{-5}, 10^{-5}]$ and the initial velocity is set equal zero. Fig. 1a illustrates that nearly all the transferred energy is used for the creation of a soliton antisoliton pair since there are no additional waves in the chain. The situation is completely different if we apply a sinusoidal driving force of the same magnitude for the same period of time and in the same region of the chain. In this case no solitons are created (see. Fig.1b) but a very complicated dynamics results due to the misfit of the driving frequency and the eigen frequency of the system (Fig.2a). This example illustrates that the response of a nonlinear system is usually very complicates whereas the response can be well predictable and simple if special aperiodic driving forces are used, sincc $u(x,t) = w(x,t)$ and $w(x,t)$ can be calculated in advance for an infinite long period of time.

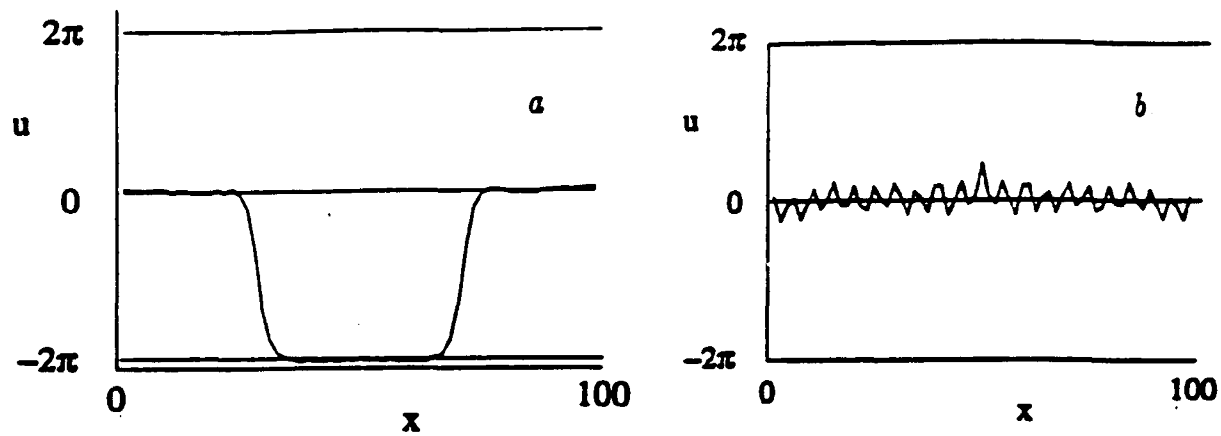

Fig. 1 The field amplitude u versus x after an aperiodic optimal stimulation (a) and after a sinusoidal stimulation (b)

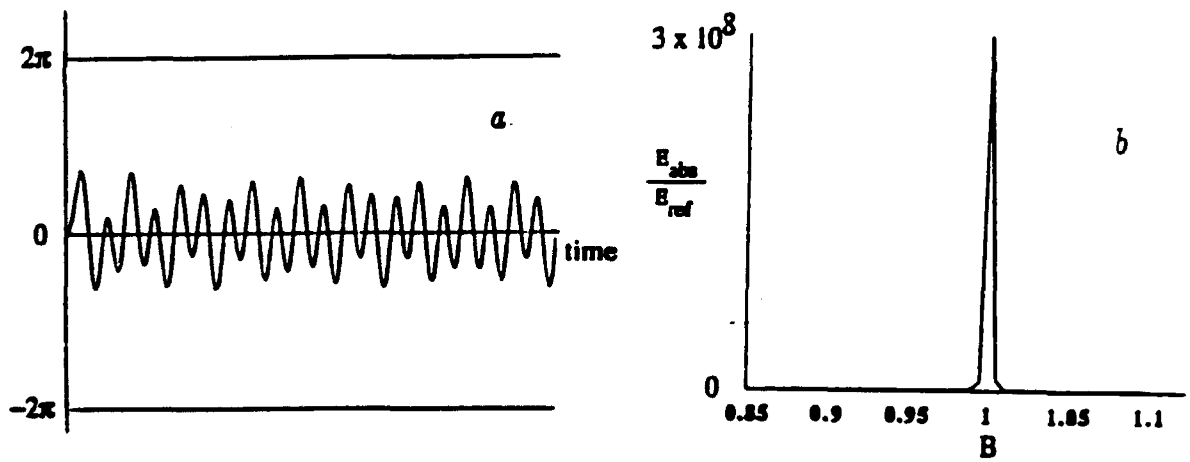

Fig. 2 The field amplitude $u\,(50, t)$ versus *time* for a sinusoidal perturbation (a) and the ratio between the reflected and the absorbed energy versus the parameter B of the model (b).

NONLINEAR RESONANCE SPECTROSCOPY

An essential condition in order to get such a simple response is to have a correct model. Otherwise u differs from w and usually the dynamics is chaotic and an essential part of the energy is reflected. Fig. 2b show the ratio R between the reflected and the absorbed energy versus B. R reaches its maximum value when the parameters of the model and the parameters of the goal dynamics coincide. In this case the response is simple and predictable for an infinite long period of time, while in all other cases including periodic perturbations a very complicated response was found. By a systematic search for the minimum of the reflected energy as a function of the parameters of the model the correct magnitude of these parameters can be determined.

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